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Numerical prediction of natural convection heat transfer in horizontal annulus

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Abstract—This paper deals with numerical prediction of natural convection heat transfer in a horizontal annulus in which the inner cylinder is hotter than the outer cylinder. A modified SIMPLE procedure is used for this purpose and it is shown that this procedure yields faster convergence. Results have been obtained for $L/D_i = 0.8$ and 0.15. This latter value is of interest in Horizontal Pressurised Heavy Water Reactors.

1. INTRODUCTION

WE CONSIDER two-dimensional steady state natural convection heat transfer in a horizontal annular gap between two concentric cylinders in which the inner cylinder is hotter than the outer one. The situation is of interest in solar concentrators, thermal storage plants, inert-gas insulated electrical cables, and in horizontal pressurised heavy water reactors (PHWRs).

Each situation however offers different values of the ratio of the gap width/inner diameter (i.e. L/D_i). Typically most previous experimental or theoretical research publications deal with $L/D_i \geqslant 0.5$. In PHWRs, the gap between the pressure tube (i.e. the inner tube) and the calendria tube (i.e. the outer tube) however is very small such that $L/D_i = 0.15$. The stable flow regime for this value is also restricted to low values (less than 10^4) of the Grashof numbers [1].

The purpose of the present investigation is to develop a correlation for $L/D_i = 0.15$. The problem derives its importance from the fact that in the event of failure of the Emergency Core Cooling System activated under loss-of-coolant accident, the moderator surrounding the calendria tube is the only heat sink available in PHWR. The rate of heat transfer between the pressure tube† and the moderator is however controlled by natural convection heat transfer through the air gap between the pressure tube and the calendria tube. For $L/D_i = 0.15$ the available correlations (experimental or semi-empirical) show poor agreement between each other.

This paper relates to numerical prediction of this natural convection heat transfer by solving governing equations for primitive variables pressure, velocities and temperature. This is in contrast with earlier investigations (see for example [2]) which use the stream function–vorticity equations. The SIMPLE algorithm of Patankar and Spalding [3] is used to solve the equations. The algorithm however is modified to enhance the rate of convergence of the numerical solutions.

Results are obtained for $L/D_i = 0.8$ and 0.15. Although the latter value is of interest, the former value is chosen to validate the program by comparing the present predictions with experimental as well as numerical results obtained earlier.

† The fuel elements are situated inside the pressure tube.

2. MATHEMATICAL PROBLEM

2.1. *Governing equations*

Figure 1 shows the geometry of interest along with the chosen coordinate system. Due to symmetry about the vertical diameter, only semi-circular annulus need be considered for solving the equations.

Consider two-dimensional steady state equations of continuity, momentum and energy transport in (r, θ) plane. When the following dimensionless parameters are used, i.e.

$$r^* = \frac{r}{r_o}, \quad Pr = \frac{\mu Cp}{K}, \quad Gr_L = \frac{g\beta(T_i - T_o)L^3}{\nu^2}$$

$$g_r = \frac{rV_r}{\nu}, \quad g_\theta = \frac{rV_\theta}{\nu}, \quad p^* = \frac{p}{\rho\nu^2/r_o^2}$$

$$T^* = \frac{T - T_o}{T_i - T_o}. \tag{1}$$

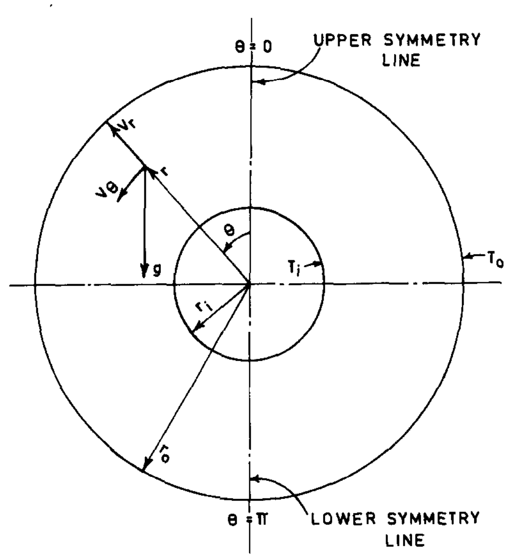

FIG. 1. The domain of interest and the coordinate system.

NOMENCLATURE

- a,b constants in the general partial differential equation (pde)
- C_N, C_S, C_E, C_W coefficients in the finite difference equation (fde)
- d source term in pde
- Gr_L Grashof number
- g_r radial velocity × radius
- g_θ tangential velocity × radius
- g acceleration due to gravity
- K_{equ} equivalent thermal conductivity
- L gap width
- p pressure
- Pr Prandtl number
- r radius
- Ra_L Rayleigh number ($Gr_L Pr$)
- S source term in fde
- T temperature
- V velocity.

Greek symbols

- α relaxation parameter
- ν kinematic viscosity
- ϕ general variable
- ω vorticity
- ψ stream function
- β coefficient of cubical expansion.

Subscripts

- i inner cylinder
- o outer cylinder
- r radial
- θ circumferential.

the governing equations can be represented in a generalized form as:

$$\underbrace{\frac{1}{r^*}\left[\frac{\partial}{\partial r^*}(a g_r \phi)+\frac{1}{r^*}\frac{\partial}{\partial \theta}(a g_\theta \phi)\right]}_{\text{convection term}} = \underbrace{\frac{1}{r^*}\left[\frac{\partial}{\partial r^*}\left(b\frac{\partial \phi}{\partial r^*}\right)+\frac{1}{r^{*2}}\frac{\partial}{\partial \theta}\left(b\frac{\partial \phi}{\partial \theta}\right)\right]}_{\text{diffusion term}} + \underbrace{d}_{\text{source term}}, \quad (2)$$

where the meanings of ϕ, a, b and d are shown in Table 1 below:

Table 1. Coefficients in equation (2)

Equation	ϕ	a	b	d
Continuity	1	1	0	0
r-momentum	q_r	$\frac{1}{r^*}$	1	d_{g_r}
θ-momentum	g_θ	$\frac{1}{r^*}$	1	d_{g_θ}
Energy	T^*	1	$\frac{r^*}{Pr}$	0

where

$$d_{g_r} = -\frac{\partial p^*}{\partial r^*} - \frac{1}{r^{*3}}\frac{\partial g_\theta}{\partial \theta} + \frac{Gr_L}{(1-r_i^*)^3} T^* \cos\theta$$

$$d_{g_\theta} = -\frac{1}{r^*}\frac{\partial p^*}{\partial \theta} + \frac{2}{r^{*3}}\frac{\partial g_r}{\partial \theta} - \frac{1}{r^{*2}}\frac{\partial g_\theta}{\partial r^*} - \frac{Gr_L}{(1-r_i^*)^3} T^* \sin\theta. \quad (3)$$

The above coupled set of equations are numerically integrated with the following boundary conditions:

Inner cylinder ($r^* = r_i^*$)

$$T^* = 1, \quad g_r = g_\theta = 0, \quad \frac{\partial p^*}{\partial r^*} = 0$$

Outer cylinder ($r^* = 1$)

$$T^* = 0, \quad g_r = g_\theta = 0, \quad \frac{\partial p^*}{\partial r^*} = 0$$

Symmetry lines ($\theta = 0$ *and* π)

$$\frac{\partial T^*}{\partial \theta} = 0, \quad g_\theta = 0, \quad \frac{\partial g_r}{\partial \theta} = \frac{\partial p^*}{\partial \theta} = 0. \quad (4)$$

After obtaining converged solutions (see Section 2.3), the stream function distribution is extracted from g_r and g_θ via the stream function equation which when written in the form of equation (2) yields:

$$\phi = \psi, \quad a = 0, \quad b = r^*, \quad d = r^*\omega, \quad (5)$$

where

$$\frac{\partial \psi}{\partial \theta} = g_r, \quad \frac{\partial \psi}{\partial r^*} = \frac{g_\theta}{r^*}$$

and

$$\omega = \frac{1}{r^*}\left[\frac{1}{r^*}\frac{\partial g_r}{\partial \theta} - \frac{\partial g_\theta}{\partial r^*}\right].$$

2.2. *Finite-difference equation*

The finite-difference equation for each ϕ is derived from equation (2) using the familiar control-volume method with reference to a staggered grid shown in Fig. 2. The method yields value of the variable ϕ at node P in terms of the values of its neighbours (N, S,

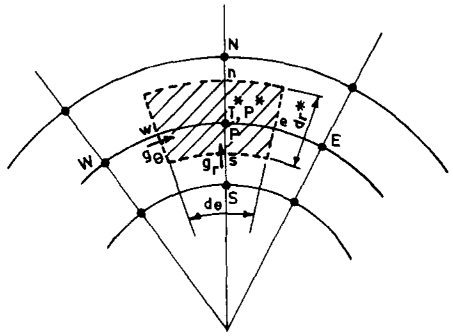

FIG. 2. Typical grid node for T^* and P^* showing staggered locations of variables.

E, W) as

$$\phi_P[C_N+C_S+C_E+C_W] = C_N\phi_N+C_S\phi_S+C_E\phi_E+C_W\theta_W+S, \quad (6)$$

where the C's represent the convective diffusive influences and S represents the effects of sources. The C's are always rendered positive since convective terms are written in upwind-difference form. Equation (6) represents a set of coupled algebraic equations for each ϕ. The set is solved iteratively by the Alternating Direction Implicit Technique.

2.3. *Modified simple algorithm*

The solution of momentum equations requires specification of pressure distribution. The SIMPLE algorithm of Patankar and Spalding [3] evaluates this pressure via a continuity equation as follows:

(i) Specify guessed pressure P^*_g and T^*.
(ii) Solve momentum equations to yield g^*_r and g^*_θ, say.

The g^*_r and g^*_θ thus obtained would not satisfy the continuity equation unless P^*_g is correctly guessed. Hence a correction to P^*_g is sought as

$$p^* = p^*_g+p', \quad (7)$$

where p' is determined via continuity equation by approximating true g_r and g_θ as

$$g_{r_s} = g^*_{r_s}+D^s(p'_S-p'_P), \quad g_{r_n} = g^*_{r_n}+D^n(p'_P-p'_N)$$

$$g_{\theta_w} = g^*_{\theta_w}+D^w(p'_W-p'_P), \quad g_{\theta_e} = g^*_{\theta_e}+D^e(p'_P-p'_E). \quad (8)$$

It may be noted that convection–diffusion influences are ignored. Also, in the source terms, terms other than those containing pressure are also ignored.

Substituting these velocities in the continuity equation, one obtains a finite-difference equation for pressure correction p' as

$$p'_P(C_N+C_S+C_E+C_W) = C_Np'_N+C_Sp'_S+C_Ep'_E+C_Wp'_W+m_P, \quad (9)$$

where

$$m_P = (g^*_{r_n}-g^*_{r_s})r^*_P\,d\theta+(g^*_{\theta_e}-g^*_{\theta_w})\,dr^*. \quad (10)$$

Solution of equation (9) by ADI method is added to p^*_g to get p^* via (7). Experience however shows that an under-relaxation is required. Thus,

$$p^* = p^*_g+\alpha p', \quad (11)$$

where α is typically 0.5–1.0.

The basis for this under-relaxation lies in ignorance of convective–diffusive and source influences in equation (8), which estimates a pressure correction which ensures satisfaction of continuity but not of momentum equations. In order to obviate this Patankar [4] proposed the SIMPLER algorithm.

Here we propose two alternatives to SIMPLE algorithm which include influences of convection, diffusion and sources in a predictor–corrector fashion.

First modification. After performing calculations as per SIMPLE the predicted velocity corrections for the whole field are stored as:

$$\begin{aligned} g'_{r_s} &= D^s(p'_S-p'_P) \\ g'_{\theta_w} &= D^w(p'_W-p'_P). \end{aligned} \quad (12)$$

Now the new values of g_r and g_θ are written as:

$$g^n_{r_s} = g_{r_s}+D^s(p''_S-p''_P)+\left[\frac{\Sigma C_i g'_{r_i}}{\Sigma C_i}+S_{g_r}(g'_r, g'_\theta)\right]_s$$

$$g^n_{\theta_w} = g_{\theta_w}+D^w(p''_W-p''_P)+\left[\frac{\Sigma C_i g'_{\theta_i}}{\Sigma C_i}+S_{g_\theta}(g'_r, g'_\theta)\right]_w \quad (13)$$

and similarly for $g^n_{r_n}$ and $g^n_{\theta_e}$.

Note that S_{g_r} and S_{g_θ} terms represent the sources of momentum equations contining g_r and g_θ in them; terms containing temperature are ignored. Now g^n_r and g^n_θ are substituted in the continuity equation and equation for p'' is obtained analogous to eqn (9), where

$$m^n_P = \left\{\left(\frac{\Sigma C_i g'_{r_i}}{\Sigma C_i}+S_{g_r}\right)_n-\left(\frac{\Sigma C_i g'_{r_i}}{\Sigma C_i}+S_{g_r}\right)_s\right\}dr^* + \left\{\left(\frac{\Sigma C_i g'_{\theta_i}}{\Sigma C_i}+S_{g_\theta}\right)_e-\left(\frac{\Sigma C_i g'_{\theta_i}}{\Sigma C_i}+S_{g_\theta}\right)_w\right\}r^*_p\,d\theta. \quad (14)$$

In the above derivation, it is noted that g_{r_s}, g_{r_n}, g_{θ_e} and g_{θ_w} in equation (13) already satisfy the continuity equation and hence do not contribute to m^n_P. Solution of p'' allows calculation of p^{*n} as:

$$p^{*n} = p^*+p''.$$

Now using equations (13), the new values of g^n_r and g^n_θ are obtained.

Second modification. In the first modification, the temperature dependent terms in sources were ignored while writing equation (13). The effect of these terms is included as follows:

After applying velocity corrections as per SIMPLE, the temperature equation is solved to yield a new temperature distribution $T^{*''}$, say. Now, the change in temperature thus predicted is stored as:

$$T' = T^{*''} - T^{*}. \tag{16}$$

Now, again equations analogous to equation (13) are written but this time the source terms S_{g_r} and S_{g_θ} are functions of g_r', g_θ' and T'. Subsequent procedure is same as before. The influence of these modifications is discussed in the next section.

In summary we note that the modifications suggested involve solution of a greater number of equations per iteration. Thus in SIMPLE 4 equations (i.e. g_r, g_θ, p', T) are solved. In the first modification 5 equations are solved (i.e. g_r, g_θ, p', p'', T) whereas in the second modification 6 equations are solved (i.e. g_r, g_θ, p', T', p'', T). Compared to SIMPLE, the first modification requires storage of two more variables (i.e. g_r' and g_θ') whereas the second modification requires storage of three more variables (i.e. g_r', g_θ' and T').

3. RESULTS AND DISCUSSION

The presence of natural convection augments the heat transfer rate compared to that which would be obtained if the heat transfer was by way of conduction alone. Hence it is convenient to define the heat transfer augmentation factor by an equivalent conductivity defined as

$$K_{\text{equ}} = \frac{\text{Heat transfer by convection}}{\text{Heat transfer by conduction}}.$$

Thus at the inner cylinder

$$\bar{K}_{\text{equ}_i} = \frac{r_i \log(r_o/r_i)}{(T_i - T_o)} \left.\frac{\overline{\partial T}}{\partial r}\right|_{r=r_i} \tag{17}$$

and at the outer cylinder

$$\bar{K}_{\text{equ}_o} = \frac{r_o \log(r_o/r_i)}{(T_i - T_o)} \left.\frac{\overline{\partial T}}{\partial r}\right|_{r=r_o} \tag{18}$$

where bars over temperature derivatives denote circumferential averages, i.e.

$$\frac{\overline{\partial T}}{\partial r} = \frac{1}{\pi}\int_0^\pi \frac{\partial T}{\partial r}\,\partial\theta. \tag{19}$$

It is quite clear that for a perfect heat balance $\bar{K}_{\text{equ}_i} = \bar{K}_{\text{equ}_o}$.

Computational details

Grid. All results for $Ra_L < 7000$ have been obtained with 27×21 ($N_\theta \times N_r$) gridlines whereas those for higher Ra_L have been obtained with 27×31 gridlines. Fine mesh sizes are provided near the inner and outer cylinders as well as near the lower and upper symmetry planes to effect accurate specifications of boundary conditions and calculation of heat flux at solid boundaries.

Convergence

The convergence of solutions has been checked both by the fractional change as well as residual source criterion. For all runs it was ensured that the maximum normalised residual source was below 10^{-5}, and that the heat imbalance was within 1.5%.

Method of computation

Solutions were first obtained for low Ra_L value, with initial guess of conduction temperature profile and zero velocities. The solutions for higher Rayleigh numbers were obtained using converged solutions for lower Rayleigh numbers. Greater under-relaxation of momentum equations was necessary at high Rayleigh numbers.

Comparison of numerical algorithms

In Table 2 below, the SIMPLE algorithm is compared in terms of computer time on CYBER 730/170 with the two modifications suggested in the previous section. Results are presented for $Ra_L = 3.29 \times 10^4$. The momentum and temperature equations were solved by Gauss–Seidel procedure whereas the pressure correction equations were solved by the ADI technique. In all cases momentum equations were under-relaxed by 50%, whereas p' equation was solved with $\alpha = 0.5$ when solving by SIMPLE. In the modifications suggested neither p' nor p'' needed any under-relaxation (i.e. $\alpha = 1.0$). No under-relaxation was applied to the temperature equation. The initial guess in each case was that corresponding to $Ra_L = 700$.

The SIMPLE algorithm is found to be extremely slow to converge and even after 1000 iterations the heat balance is only 2.7%. The modifications suggested, although they involve solution of a greater number of equations per iteration, are found to be nearly 4 times faster than SIMPLE.

Confirmatory calculations for $L/D_i = 0.8$, $Pr = 0.7$

Figure 3 shows the comparison of present predictions of mean conductivity with numerical predictions and experimental correlation of Kuehn and Goldstein [2], Grigull and Hauf [6] and Boyd [5]. The

Table 2. Comparison of numerical algorithms

Algorithm	Number of iterations	CPU Sec.	Heat balance (%)
SIMPLE	1000	3080	2.7
First modification	170	765	0.17
Second modification	120	653	0.7

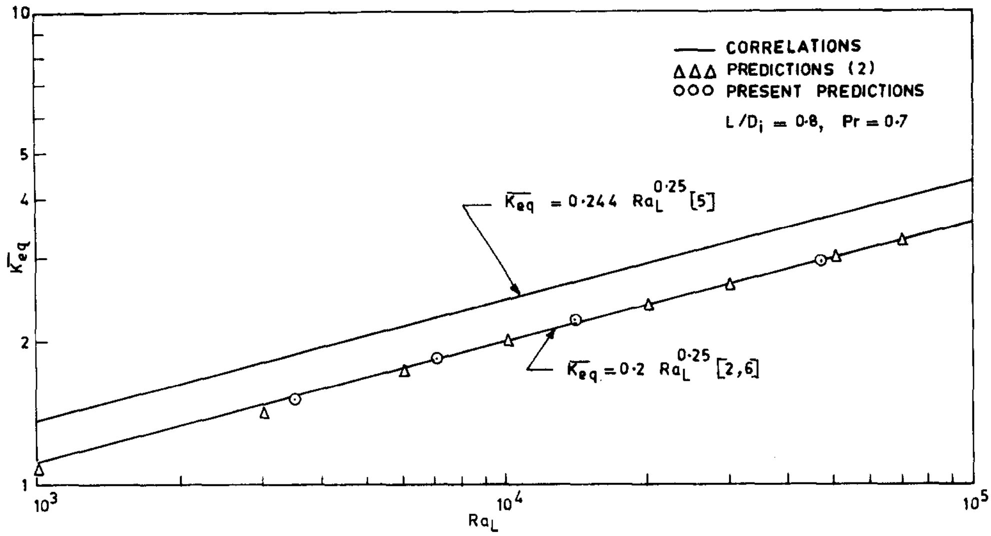

FIG. 3. Equivalent conductivity as a function of Rayleigh number.

agreement with numerical predictions of Ref. [2] and experimental correlations of Refs. [2, 6] is excellent. The semi-empirical (boundary layer type) analysis of Boyd [5] however over-predicts the data. This is probably because of the failure of the analysis to properly account for the recirculation region near the top and bottom axes of symmetry.

Figure 4 shows the comparison of local conductivities at inner and outer cylinders with predictions of Ref. [2]. Except near the lower symmetry line at the outer wall the agreement is excellent. The small disagreement is likely due to the fact that Kuehn and Goldstein have used coarser grid than used in the present investigation.

The presently computed temperature profiles also show good agreement ($\pm 5\%$) with measurements of [2] as shown in Fig. 5. Similar conclusion can be drawn from Fig. 6 where numerically computed values of angular velocity from [2] are compared with present predictions.

Figure 7 shows the comparison between the present prediction and those of [2] as it relates to stream function and temperature contours. The figure clearly brings out the fact that natural convection causes recirculation of the fluid with warm fluid rising up near the inner cylinder and descending near the outer one. It may be noted that prediction of [2] are for $Ra_L = 5 \times 10^4$.

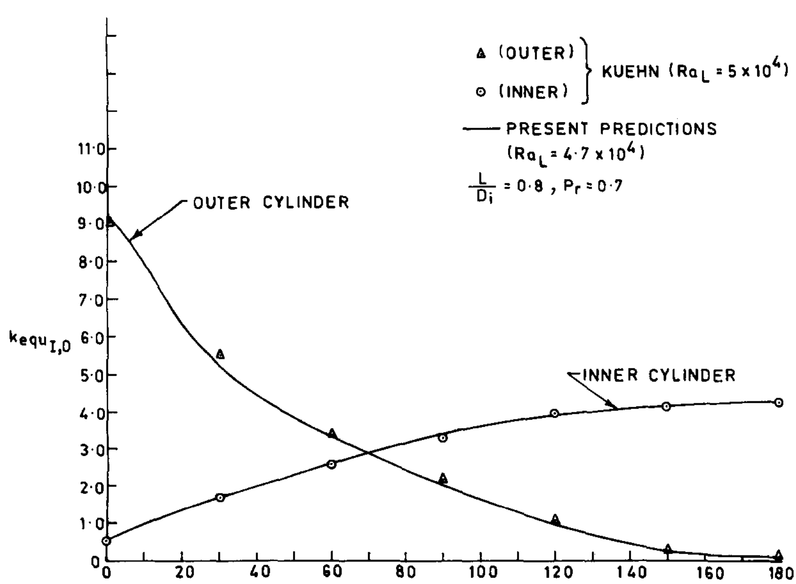

FIG. 4. Comparison of local equivalent conductivities at inner and outer walls.

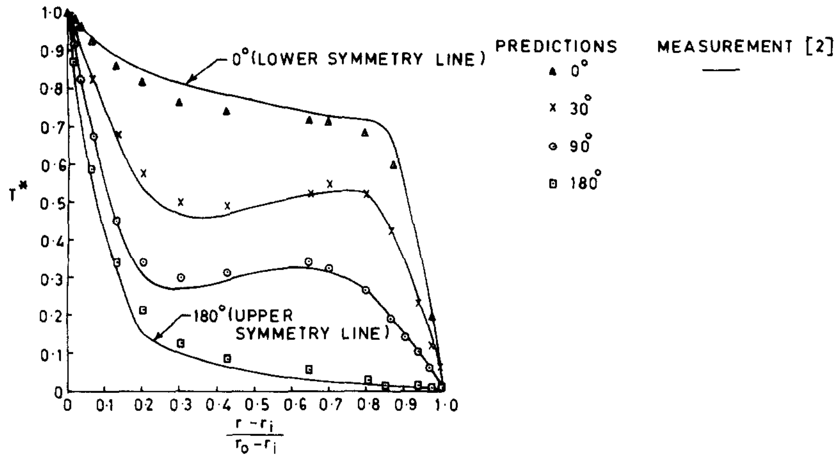

FIG. 5. Comparison of predicted and measured temperature profiles for $Ra_L = 4.73\times10^4$, $Pr = 0.7$, $L/D_i = 0.8$.

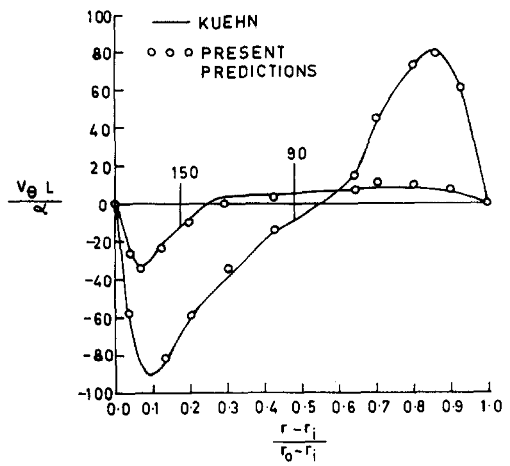

FIG. 6. Distribution of angular velocity for $Ra_L = 4.73\times10^4$, $Pr = 0.7$, $L/D_i = 0.8$.

Predictions for $L/D_i = 0.15$, $Pr = 0.7$

Figure 8 shows the variation of mean conductivities with Ra_L. The data can be well correlated by

$$\bar{K}_{equ} = 0.65 Ra_L^{0.08}.$$

The figure also compares correlations of Boyd [5] and Grigull and Hauf [6]. It is seen that both these correlations considerably over-predict the presently computed data. In fact it is clear that these correlations are not suitable for low values of Ra_L and L/D_i. It may be mentioned however that for $L/D_i = 0.1$, $Pr = 0.7$ and $Ra_L = 10^4$, Kuehn and Goldstein calculate $\bar{K}_{equ} = 1.272$. This is the only value computed by them and the order of magnitude is in accord with the present predictions.

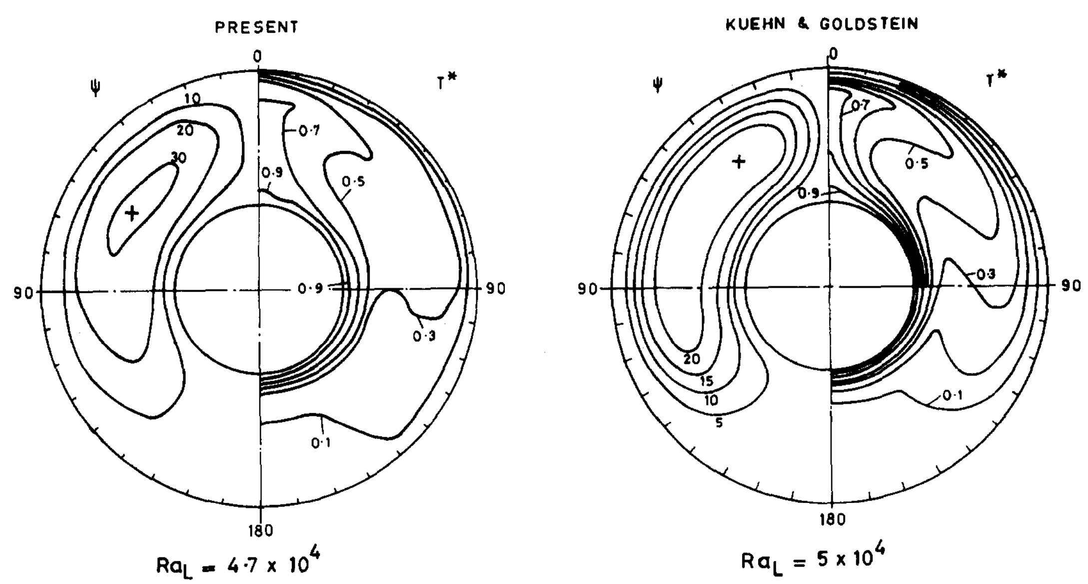

FIG. 7. Predicted and measured stream function and temperature profiles.

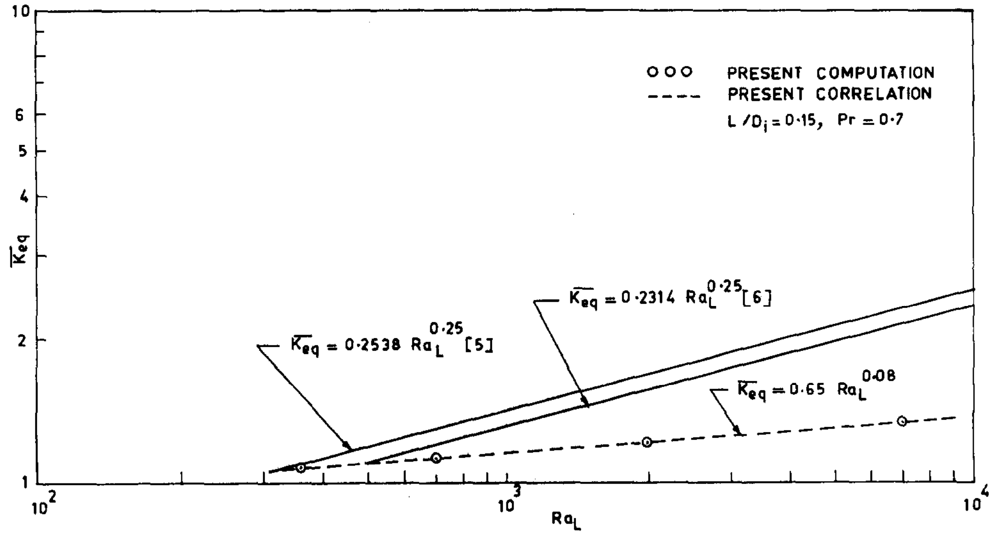

FIG. 8. Equivalent conductivity as a function of Rayleigh number.

CONCLUSIONS

(1) In this paper numerical predictions for natural convection heat transfer in horizontal annulus have been obtained for gap width/internal diameter (i.e. L/D_i) ratio of 0.8 and 0.15 for $Pr = 0.7$ using a modified form of the SIMPLE numerical procedure of Patankar and Spalding [3].

(2) The modified procedure has been found to effect faster convergence as it accounts for effects of convection and diffusion as well as some of the source terms in estimation of pressure correction.

(3) Results obtained for $L/D_i = 0.8$ have shown excellent agreement with previous numerical as well as experimental data.

(4) In horizontal PHWRs $L/D_i = 0.15$ is of interest. It is shown that all the previously generalised correlations not only do not match between themselves but also over-predict the presently computed heat transfer rates. The following correlation is recommended for $L/D_i = 0.15$, $Pr = 0.7$, $300 < Ra_L < 7000$: $\bar{K}_{equ} = 0.65 Ra_L^{0.08}$.

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REFERENCES

1. E. H. Bishop, C. T. Carley and R. E. Powe, Natural convective oscillatory flow in cylindrical annuli, *Int. J. Heat Mass Transfer* **11**, 1741–1752 (1968).
2. T. H. Kuehn and R. J. Goldstein, An experimental and theoretical study of natural convection in the annulus between horizontal concentric cylinders, *J. Fluid Mech.* **74**, 695–719 (1976).
3. S. V. Patankar and D. B. Spalding, A calculation procedure for heat mass and momentum transfer in three-dimensional parabolic flows, *Int. J. Heat Mass Transfer* **15**, 1787–1799 (1972).
4. S. V. Patankar, *Numerical Fluid Flow and Heat Transfer*. Academic Press, New York (1981).
5. R. D. Boyd, A unified theory for correlating steady laminar convective heat transfer data for horizontal annuli, *Int. J. Heat Mass Transfer* **24**, 1545–1548 (1981).
6. U. Grigull and W. Hauf, Natural convection in horizontal cylindrical annuli. *Proc. Third Int. Heat Transfer Conf.* **2**, 182–195 (1966).

PREVISION NUMERIQUE DE LA CONVECTION THERMIQUE NATURELLE DANS UN ESPACE ANNULAIRE HORIZONTAL

Résumé—On traite du calcul numérique de la convection thermique naturelle dans un espace annulaire horizontal, dans lequel un cylindre intérieur est plus chaud que le cylindre extérieur. Une procédure SIMPLE modifiée est utilisée dans ce but et on montre qu'elle fournit une convergence rapide. Des résultats sont obtenus pour L/D_i = 0,8 et 0,15. La dernière valeur présente de l'intérêt dans les réacteurs à eau lourde pressurisée horizontaux.

NUMERISCHE BERECHNUNG DER NATÜRLICHEN KONVEKTION IN HORIZONTALEN RINGRÄUMEN

Zusammenfassung—Der Artikel behandelt die numerische Berechnung des Wärmeübergangs bei natürlicher Konvektion in einem horizontalen Ringraum, wobei der innere Zylinder wärmer ist als der äußere. Eine modifizierte Form der SIMPLE-Methode wird benutzt, und es wird gezeigt, daß diese Methode schneller konvergiert. Ergebnisse wurden für Werte von $L/D_i = 0{,}8$ und 0,15 berechnet, letzterer Wert ist von Interesse in waagerechten mit schwerem Wasser betriebenen Druckwasserreaktoren.

РАСЧЕТ ЕСТЕСТВЕННОКОНВЕКТИВНОГО ТЕПЛООБМЕНА В ГОРИЗОНТАЛЬНОМ КОЛЬЦЕВОМ СЛОЕ

Аннотация—Рассчитан естественноконвективный теплообмен в горизонтальном кольцевом слое, причем внутренний цилиндр теплее наружного. Используется модифицированная методика SIMPLE; показано, что она дает более быструю сходимость. Результаты получены для $L/D_i = 0{,}8$ и 0,15. Последнее значение представляет интерес для горизонтальных герметизированных реакторов на тяжелой воде.